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THE EQUIVALENCE OF TWO APPROACHES OF SEIBERG-WITTEN EQUATIONS IN 8-DIMENSION

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Abstract. Seiberg−Witten equations which are formed by Dirac equation and Curvature−equation, have some generalizations on 8−dimensional manifold [1, 3, 5]. In this paper we consider the $Spin^c$−structure which was given in [1]. Then by using this $Spin^c$−structure, we examine the curvature equations which were given in [1, 3]. Finally we show the equivalence between them.

1. Definition and Notation

A complex vector bundle S can be constructed by a given $Spin^c$ representation $\kappa_n : Spin^c(n) \to Aut(\Delta_n)$ and denoted by $S = P_{Spin^c(n)} \times_{\kappa_n} \Delta n$. Also this complex vector bundle is called spinor bundle for a given $Spin^c$−structure on M. Moreover sections of S are called spinor fields on M. If the dimension of M is even, then spinor bundles splits into two pieces $S = S^+ \otimes S^-$ [4]. $\kappa_n : \mathbb{R}^n \to End(\Delta_n)$ is a linear map satisfying the following conditions:

$$\kappa_n(v)^* + \kappa_n(v) = 0, \qquad \kappa_n(v)^*\kappa_n(v) = |v|^2\mathbb{I}.$$

for any $v \in \mathbb{R}^n$. Let $\{e_1, e_2, ..., e_n\}$ be orthonormal frame on open subset $U \subset M$. Then

$$\begin{array}{rcl} \rho : \Lambda^2(T^*M) & \to & End(S) \\ \eta = \sum\limits_{i<j} \eta_{ij} e^i \wedge e^j & \to & \rho(\eta) = \sum\limits_{i<j} \eta_{ij}\kappa(e_i)\kappa(e_j). \end{array}$$

can be defined on the frames by extending map $\kappa : TM \to End(S)$ of κ_n. Also ρ can be extended to complex valued 2−forms such that

$$\rho : \Lambda^2(T^*M) \otimes \mathbb{C} \to End(S).$$

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The half−spinor bundle $S^{\pm}$ are invariant under $\rho(\eta)$ for all $\eta \in \Lambda^2\big(T^*(M) \otimes \mathbb{C}\big)$. That is,

$$\begin{aligned} \rho(\eta)(\psi) &\in S^+, \qquad \forall \psi \in S^+ \\ \rho(\eta)(\psi) &\in S^-, \qquad \forall \psi \in S^-. \end{aligned}$$

Then, we obtain the following maps by restriction $\rho^+(\eta) = \rho(\eta)\Big|_{S^+}$, $\rho^-(\eta) = \rho(\eta)\Big|_{S^-}$. In this case

$$\rho^+ : \Lambda^2(T^*M) \otimes \mathbb{C} \to End(S^+)$$

is expressed as follows:

$$\rho^+(\eta) = \rho^+\big(\sum_{i<j} \eta_{ij} e^i \wedge e^j\big) = \sum_{i<j} \eta_{ij} \kappa(e_i)\kappa(e_j)\big).$$

A connection ∇^A on S, which is called spinor covariant derivative operator, is obtained by using an $i\mathbb{R}-$valued 1−form $A \in \Omega(M, i\mathbb{R})$ and the Levi−Civita connection ∇ on M. At this point the definition of Dirac operator $D_A : \Gamma(S) \to \Gamma(S)$ can be given by

$$D_A(\Psi) = \sum_{i=1}^{n} \kappa(e_i)\nabla^A_{e_i}\Psi.$$

where $\Psi \in \Gamma(S)$, $\{e_1, e_2, ..., e_n\}$ is any positively oriented local orthonormal frame of TM see [4].

2. Seiberg-Witten Equations on 4−Manifolds :

Let us consider the Hodge star operator

$$* : \Lambda^k(M) \to \Lambda^{4-k}(M)$$

and in particular, its action on the 2−forms

$$* : \Lambda^2(M) \to \Lambda^2(M).$$

Since $*^2 = id$, $*$ induces a splitting of $\Lambda^2(M) = \Lambda^2_+(M) \oplus \Lambda^2_-(M)$, where $\Lambda^2_+(M) = \{\eta \in \Lambda^2(M) | * \eta = \eta\}\})$ and $\Lambda^2_-(M) = \{\eta \in \Lambda^2(M) | * \eta = -\eta\}$ indicate the space of self−dual and anti−self−dual 2−forms respectively.

The projection of a 2−form $\eta \in \Lambda^2(M)$ onto the subspace $\Lambda^2_+(M)$ is called the self−dual part of η and we denote it by η^+, similarly the projection of η onto the subspace Λ^- is called the anti−self−dual part of η and we denote it by η^-.
Also this decomposition can be extended to the $i\mathbb{R}$ valued 2−form space $\Omega^2(M, i\mathbb{R})$ and expressed as in follow:

$$\Omega^{2,+}(M, i\mathbb{R}) \oplus \Omega^{2,-}(M, i\mathbb{R}).$$

If $F_A \in \Omega^2(M, i\mathbb{R})$, then it can be written as $F_A = F_A^+ + F_A^-$, with $F_A^+ \in \Omega^{2,+}(M, i\mathbb{R})$ and $F_A^- \in \Omega^{2,-}(M, i\mathbb{R})$.

The Seiberg−Witten equations on 4− dimensional $Spin^c$ manifold can be expressed as follow:

(1) $D_A\Psi = 0$

(2) $\rho^+\left(F_A^+\right) = \left(\Psi\Psi^*\right)_0$

where $\left(\Psi\Psi^*\right)_0$ is the tracefree part of $\Psi\Psi^*$ [2]. The first part of these equations is called Dirac equation and the other is called Curvature equation.

In the following section, Seiberg−Witten equations were constructed on 8−dimensional manifold by using self−duality which is given in [1, 3].

3. Seiberg-Witten Equations on 8−manifolds

Let us consider 8−dimensional manifold M with structure group Spin(7) where $Spin(7)$ is a subgroup of $SO(8)$. In this case, there is a fundemantal 4−form Φ on M, which is nonzero everywhere. With the aid of this 4−form, $\Omega^2(M)$ splits up into

$$\Omega^2(M) = \Omega^2_7(M) \oplus \Omega^2_{21}(M),$$

where

$$\Omega^2_7(M) = \{\omega \in \Omega^2(M) | *(\Phi \wedge \omega) = 3\omega\}$$

and

$$\Omega^2_{21}(M) = \{\omega \in \Omega^2(M) | *(\Phi \wedge \omega) = -\omega\}.$$

In this study we considered $\Omega^2_7(M)$ as the space of self− dual 2−forms which is given in [1, 3].

For the i$\mathbb{R}$ valued 2−forms decomposition can be written as

$$\Omega^2(M, i\mathbb{R}) = \Omega^{2,+}(M, i\mathbb{R}) \oplus \Omega^{2,-}(M, i\mathbb{R}).$$

F_A is an element of $\Omega^2(M, i\mathbb{R})$, so that we have $F_A = F_A^+ + F_A^-$, with $F_A^+ \in \Omega^{2,+}(M, i\mathbb{R})$ and $F_A^- \in \Omega^{2,-}(M, i\mathbb{R})$. If F_A^+ more explicitly expressed:

$$F_A^+ = Proj_{\Omega^2_7(M,i\mathbb{R})} F_A.$$

With the aid of $\rho^+ : \Omega^2(M, i\mathbb{R}) \to End(S^+)$, let the image of $\Omega^2_7(M, i\mathbb{R})$ be $\rho^+\left(\Omega^2_7(M, i\mathbb{R})\right) = W^{'} \subset End(S^+)$ and for $\Psi \in \Gamma(S)$, let the projection of $\Psi\Psi^*$ on W' be $(\Psi\Psi^*)^+ = Proj_{W'}(\Psi\Psi^*)$. Then the Seiberg−Witten equations on 8−dimensional manifold with $Spin(7)$−structure is given as follow [1]:

$$\begin{aligned} D_A^+(\Psi) &= 0 \\ \rho^+\left(F_A^+\right) &= \left(\Psi\Psi^*\right)^+. \end{aligned} \tag{3.1}$$

On 8−dimensional manifold with $Spin(7)$−structure for $M = \mathbb{R}^8$ the Seiberg−Witten equations were obtained as in the following [1].

3.1. Some Local Discussions.

In 8−dimensional manifolds, the vector space of complex 8−spinors is $\Delta_8 = \mathbb{C}^{16}$. Its know that $\mathbb{C}l_8 \cong End(\Delta_8)$. The Spin^c structure is given by

$$\begin{aligned} \kappa_8 : \mathbb{C}l_8 &\to End(\Delta_8) \\ e_i &\mapsto \kappa_8(e_i) = \begin{bmatrix} 0 & \gamma(e_i) \\ -\gamma(e_i)^* & 0 \end{bmatrix} \end{aligned}$$

where $e_i,\ i = 1, ..., 8$ is the stanndart basis for $\mathbb{R}^8$ and $\gamma : \mathbb{R}^8 \to End(\mathbb{C}^8)$ is defined on generators $\{e_1, e_2, e_3, ..., e_8\}$ by the followings [1]:

$$\gamma(e_1) = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \gamma(e_2) = \begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{bmatrix}$$

$$\gamma(e_3) = \begin{bmatrix} 0&1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0 \end{bmatrix} \gamma(e_4) = \begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0\\ -1&0&0&0&1&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&1&0&0&0&0&0 \end{bmatrix}$$

$$\gamma(e_5) = \begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&-1&0&0 \end{bmatrix} \gamma(e_6) = \begin{bmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{bmatrix}$$

$$\gamma(e_7) = \begin{bmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0 \end{bmatrix} \gamma(e_8) = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0 \end{bmatrix}.$$

For $i = 2, 3, ..., 8$ these γ_i matrices are the image of the generators of $\mathbb{C}l_7$ under an explicit isomorphism $\mathbb{C}l_7 \cong End(\mathbb{C}^8) \oplus End(\mathbb{C}^8)$ which is obtained from the periodicity relation $\mathbb{C}l_{n+2} \cong \mathbb{C}l_n \otimes_{\mathbb{C}} \mathbb{C}$ [4].

The associated connection on the line bundle P_{S^1} is the connection $1-$form and represented by

$$A = \sum_{i=1}^{8} A_i dx^i \in \Omega(\mathbb{R}^8, i\mathbb{R})$$

and its curvature $2-$form is given by

$$F_A = dA = \sum_{i<j}^{8} F_{ij} dx^i \wedge dx^j \in \Omega^2(\mathbb{R}^8, i\mathbb{R})$$

where $F_{ij} = \left(\frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j} \right)$ for $i, j = 1, 2, .., 8$.

The Spin^c-connection ∇^A on $\mathbb{R}^8$ is given by

$$\nabla^A_{e_j} \Psi = \frac{\partial \Psi}{\partial x_j} + \frac{1}{2} A_j \Psi,$$

where $\Psi \in \Gamma(S^+)$. According to these explicit form of the first equation $D_A^+\Psi = 0$ is

$$\begin{array}{ll}
- & \frac{\partial}{\partial x_1}\psi_1 + \frac{\partial}{\partial x_3}\psi_2 + \frac{\partial}{\partial x_5}\psi_3 + \frac{\partial}{\partial x_7}\psi_4 + \frac{\partial}{\partial x_2}\psi_5 + \frac{\partial}{\partial x_4}\psi_6 + \frac{\partial}{\partial x_6}\psi_7 + \frac{\partial}{\partial x_8}\psi_8 \\
+ & \frac{1}{2}\Big(-\psi_1 A_1 + \psi_5 A_2 + \psi_2 A_3 + \psi_6 A_4 + \psi_3 A_5 + \psi_7 A_6 + \psi_4 A_7 + \psi_8 A_8 \Big) = 0 \\
- & \frac{\partial}{\partial x_3}\psi_1 - \frac{\partial}{\partial x_1}\psi_2 + \frac{\partial}{\partial x_7}\psi_3 - \frac{\partial}{\partial x_5}\psi_4 - \frac{\partial}{\partial x_4}\psi_5 + \frac{\partial}{\partial x_2}\psi_6 + \frac{\partial}{\partial x_6}\psi_8 - \frac{\partial}{\partial x_8}\psi_7 \\
+ & \frac{1}{2}\Big(-\psi_2 A_1 + \psi_6 A_2 - \psi_1 A_3 - \psi_5 A_4 - \psi_4 A_5 + \psi_8 A_6 + \psi_3 A_7 - \psi_7 A_8 \Big) = 0 \\
- & \frac{\partial}{\partial x_5}\psi_1 - \frac{\partial}{\partial x_7}\psi_2 - \frac{\partial}{\partial x_1}\psi_3 + \frac{\partial}{\partial x_3}\psi_4 - \frac{\partial}{\partial x_6}\psi_5 + \frac{\partial}{\partial x_2}\psi_7 - \frac{\partial}{\partial x_4}\psi_8 + \frac{\partial}{\partial x_8}\psi_6 \\
+ & \frac{1}{2}\Big(-\psi_3 A_1 + \psi_7 A_2 + \psi_4 A_3 - \psi_8 A_4 - \psi_1 A_5 - \psi_5 A_6 - \psi_2 A_7 + \psi_6 A_8 \Big) = 0 \\
- & \frac{\partial}{\partial x_7}\psi_1 + \frac{\partial}{\partial x_5}\psi_2 - \frac{\partial}{\partial x_3}\psi_3 - \frac{\partial}{\partial x_1}\psi_4 - \frac{\partial}{\partial x_6}\psi_6 + \frac{\partial}{\partial x_4}\psi_7 + \frac{\partial}{\partial x_2}\psi_8 - \frac{\partial}{\partial x_8}\psi_5 \\
+ & \frac{1}{2}\Big(-\psi_4 A_1 + \psi_8 A_2 - \psi_3 A_3 + \psi_7 A_4 + \psi_2 A_5 - \psi_6 A_6 - \psi_1 A_7 - \psi_5 A_8 \Big) = 0 \\
- & \frac{\partial}{\partial x_2}\psi_1 + \frac{\partial}{\partial x_4}\psi_2 + \frac{\partial}{\partial x_6}\psi_3 - \frac{\partial}{\partial x_1}\psi_5 - \frac{\partial}{\partial x_3}\psi_6 - \frac{\partial}{\partial x_5}\psi_7 - \frac{\partial}{\partial x_7}\psi_8 + \frac{\partial}{\partial x_8}\psi_4 \\
+ & \frac{1}{2}\Big(-\psi_5 A_1 - \psi_1 A_2 - \psi_6 A_3 - \psi_7 A_5 + \psi_3 A_6 - \psi_8 A_7 + \psi_4 A_8 + \psi_2 A_4 \Big) = 0 \\
- & \frac{\partial}{\partial x_4}\psi_1 - \frac{\partial}{\partial x_2}\psi_2 + \frac{\partial}{\partial x_6}\psi_4 + \frac{\partial}{\partial x_3}\psi_5 - \frac{\partial}{\partial x_1}\psi_6 - \frac{\partial}{\partial x_7}\psi_7 + \frac{\partial}{\partial x_5}\psi_8 - \frac{\partial}{\partial x_8}\psi_3 \\
+ & \frac{1}{2}\Big(-\psi_6 A_1 - \psi_2 A_2 + \psi_5 A_3 - \psi_1 A_4 + \psi_8 A_5 + \psi_4 A_6 - \psi_7 A_7 - \psi_3 A_8 \Big) = 0 \\
- & \frac{\partial}{\partial x_6}\psi_1 - \frac{\partial}{\partial x_2}\psi_3 - \frac{\partial}{\partial x_4}\psi_4 + \frac{\partial}{\partial x_5}\psi_5 + \frac{\partial}{\partial x_7}\psi_6 - \frac{\partial}{\partial x_1}\psi_7 - \frac{\partial}{\partial x_3}\psi_8 + \frac{\partial}{\partial x_8}\psi_2 \\
+ & \frac{1}{2}\Big(-\psi_7 A_1 - \psi_3 A_2 - \psi_8 A_3 - \psi_4 A_4 + \psi_5 A_5 - \psi_1 A_6 + \psi_6 A_7 + \psi_2 A_8 \Big) = 0 \\
- & \frac{\partial}{\partial x_6}\psi_2 + \frac{\partial}{\partial x_4}\psi_3 - \frac{\partial}{\partial x_2}\psi_4 + \frac{\partial}{\partial x_7}\psi_5 - \frac{\partial}{\partial x_5}\psi_6 + \frac{\partial}{\partial x_3}\psi_7 - \frac{\partial}{\partial x_1}\psi_8 - \frac{\partial}{\partial x_8}\psi_1 \\
+ & \frac{1}{2}\Big(-\psi_8 A_1 - \psi_4 A_2 + \psi_7 A_3 + \psi_3 A_4 - \psi_6 A_5 - \psi_2 A_6 + \psi_5 A_7 - \psi_1 A_8 \Big) = 0.
\end{array}$$

In the following section, the curvature equation on $8-$dimensional manifod is obtained according to [1]

3.1.1. *Curvature Equation on* $\mathbb{R}^8$ *:* The second part of Seiberg$-$Witten equation, which is called curvature equation and denoted by $\rho^+\big(F_A^+\big) = \big(\Psi\Psi^*\big)^+$, is described by the orthogonal bases of $\Omega_7^2\big(\mathbb{R}^8, i\mathbb{R}\big)$, which are given in the following [1]:

$$\begin{aligned}
f_1 &= dx_1 \wedge dx_5 + dx_2 \wedge dx_6 + dx_3 \wedge dx_7 + dx_4 \wedge dx_8 \\
f_2 &= dx_1 \wedge dx_2 + dx_3 \wedge dx_4 - dx_5 \wedge dx_6 - dx_7 \wedge dx_8 \\
f_3 &= dx_1 \wedge dx_6 - dx_2 \wedge dx_5 - dx_3 \wedge dx_8 + dx_4 \wedge dx_7 \\
f_4 &= dx_1 \wedge dx_3 - dx_2 \wedge dx_4 - dx_5 \wedge dx_7 + dx_6 \wedge dx_8 \\
f_5 &= dx_1 \wedge dx_7 + dx_2 \wedge dx_8 - dx_3 \wedge dx_5 - dx_4 \wedge dx_6 \\
f_6 &= dx_1 \wedge dx_4 + dx_2 \wedge dx_3 - dx_5 \wedge dx_8 - dx_6 \wedge dx_7 \\
f_7 &= dx_1 \wedge dx_8 - dx_2 \wedge dx_7 + dx_3 \wedge dx_6 - dx_4 \wedge dx_5.
\end{aligned}$$

According to these orthogonal bases $\rho^+\big(F_A^+\big) = (\Psi\Psi^*)^+$ is equivalent to the equation

$$\rho^+\big(F_A^+\big) \quad = \quad \sum_{i=1}^{7} \frac{\big\langle \rho^+(f_i), (\Psi\Psi^*) \big\rangle}{\big\langle \rho^+(f_i), \rho^+(f_i) \big\rangle}.\rho^+(f_i)$$

and the more explicit form of the equation is

$$F_{15}+F_{26}+F_{37}+F_{48}=\tfrac{1}{4}(\psi_1\bar{\psi}_3-\psi_3\bar{\psi}_1-\psi_2\bar{\psi}_4+\psi_4\bar{\psi}_2-\psi_5\bar{\psi}_7+\psi_7\bar{\psi}_5-\psi_4\bar{\psi}_8+\psi_8\bar{\psi}_6)$$
$$F_{12}+F_{34}-F_{56}-F_{78}=\tfrac{1}{4}(\psi_1\bar{\psi}_5-\psi_5\bar{\psi}_1-\psi_2\bar{\psi}_6+\psi_6\bar{\psi}_2+\psi_3\bar{\psi}_7-\psi_7\bar{\psi}_3+\psi_4\bar{\psi}_8-\psi_8\bar{\psi}_4)$$
$$F_{16}-F_{25}-F_{38}+F_{47}=\tfrac{1}{4}(\psi_1\bar{\psi}_7-\psi_7\bar{\psi}_1+\psi_2\bar{\psi}_8-\psi_8\bar{\psi}_2-\psi_3\bar{\psi}_5+\psi_5\bar{\psi}_3+\psi_4\bar{\psi}_6-\psi_6\bar{\psi}_4)$$
$$F_{13}-F_{24}-F_{57}+F_{68}=\tfrac{1}{4}(\psi_1\bar{\psi}_2-\psi_2\bar{\psi}_1+\psi_3\bar{\psi}_4-\psi_4\bar{\psi}_3+\psi_5\bar{\psi}_6-\psi_6\bar{\psi}_5-\psi_7\bar{\psi}_8+\psi_8\bar{\psi}_7)$$
$$F_{17}+F_{28}-F_{35}-F_{46}=\tfrac{1}{4}(\psi_1\bar{\psi}_4-\psi_4\bar{\psi}_1+\psi_2\bar{\psi}_3-\psi_3\bar{\psi}_2-\psi_5\bar{\psi}_8+\psi_8\bar{\psi}_5+\psi_6\bar{\psi}_7-\psi_7\bar{\psi}_6)$$
$$F_{14}+F_{23}-F_{58}-F_{67}=\tfrac{1}{4}(\psi_6\bar{\psi}_1-\psi_1\bar{\psi}_6-\psi_2\bar{\psi}_5+\psi_5\bar{\psi}_2-\psi_3\bar{\psi}_8+\psi_8\bar{\psi}_3+\psi_4\bar{\psi}_7-\psi_7\bar{\psi}_4)$$
$$F_{18}-F_{27}-F_{36}-F_{45}=\tfrac{1}{4}(\psi_1\bar{\psi}_8-\psi_8\bar{\psi}_1-\psi_2\bar{\psi}_7+\psi_7\bar{\psi}_2-\psi_3\bar{\psi}_6+\psi_6\bar{\psi}_3-\psi_4\bar{\psi}_5-\psi_5\bar{\psi}_4).$$

In the following section, by using alternative method Seiberg−Witten equations were obtained. Then equivalence between 3.1 and 3.2 holds.

3.2. **Alternative method for the curvature equation.** Let $\Psi \in \Gamma(S^+)$ then $i\mathbb{R}$ valued 2−form $\sigma(\psi)$ is defined by the following formula:

$$\sigma(\psi)(X,Y) = \langle X\cdot Y\cdot\psi,\psi\rangle + \langle X,Y\rangle|\psi|^2,$$

where $X, Y \in \chi(M)$ [4]. Then the Seiberg−Witten equation on 8−dimensional Spin^c manifold can be expressed as follow:

$$D_A\Psi = 0,$$
$$F_A^+ = \frac{1}{8}\sigma(\Psi)^+, \tag{3.2}$$

where F_A^+ is the self−dual part of the curvature F_A and $\sigma(\Psi)^+$ is the projection of $\sigma(\Psi)$ on to $\Omega^2_7(M, i\mathbb{R})$.

In alternative method there is no difference other than curvature equation. According to new method the equation set of curvature equation is obtained as follow:

$$F_{15}+F_{26}+F_{37}+F_{48}=\tfrac{1}{4}(\psi_1\bar{\psi}_3-\psi_3\bar{\psi}_1-\psi_2\bar{\psi}_4+\psi_4\bar{\psi}_2-\psi_5\bar{\psi}_7+\psi_7\bar{\psi}_5-\psi_4\bar{\psi}_8+\psi_8\bar{\psi}_6)$$
$$F_{12}+F_{34}-F_{56}-F_{78}=\tfrac{1}{4}(\psi_1\bar{\psi}_5-\psi_5\bar{\psi}_1-\psi_2\bar{\psi}_6+\psi_6\bar{\psi}_2+\psi_3\bar{\psi}_7-\psi_7\bar{\psi}_3+\psi_4\bar{\psi}_8-\psi_8\bar{\psi}_4)$$
$$F_{16}-F_{25}-F_{38}+F_{47}=\tfrac{1}{4}(\psi_1\bar{\psi}_7-\psi_7\bar{\psi}_1+\psi_2\bar{\psi}_8-\psi_8\bar{\psi}_2-\psi_3\bar{\psi}_5+\psi_5\bar{\psi}_3+\psi_4\bar{\psi}_6-\psi_6\bar{\psi}_4)$$
$$F_{13}-F_{24}-F_{57}+F_{68}=\tfrac{1}{4}(\psi_1\bar{\psi}_2-\psi_2\bar{\psi}_1+\psi_3\bar{\psi}_4-\psi_4\bar{\psi}_3+\psi_5\bar{\psi}_6-\psi_6\bar{\psi}_5-\psi_7\bar{\psi}_8+\psi_8\bar{\psi}_7)$$
$$F_{17}+F_{28}-F_{35}-F_{46}=\tfrac{1}{4}(\psi_1\bar{\psi}_4-\psi_4\bar{\psi}_1+\psi_2\bar{\psi}_3-\psi_3\bar{\psi}_2-\psi_5\bar{\psi}_8+\psi_8\bar{\psi}_5+\psi_6\bar{\psi}_7-\psi_7\bar{\psi}_6)$$
$$F_{14}+F_{23}-F_{58}-F_{67}=\tfrac{1}{4}(\psi_6\bar{\psi}_1-\psi_1\bar{\psi}_6-\psi_2\bar{\psi}_5+\psi_5\bar{\psi}_2-\psi_3\bar{\psi}_8+\psi_8\bar{\psi}_3+\psi_4\bar{\psi}_7-\psi_7\bar{\psi}_4)$$
$$F_{18}-F_{27}-F_{36}-F_{45}=\tfrac{1}{4}(\psi_1\bar{\psi}_8-\psi_8\bar{\psi}_1-\psi_2\bar{\psi}_7+\psi_7\bar{\psi}_2-\psi_3\bar{\psi}_6+\psi_6\bar{\psi}_3-\psi_4\bar{\psi}_5-\psi_5\bar{\psi}_4)..$$

According to above data equivalence between 3.1 and 3.2 hold.

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